J Seismol
DOI 10.1007/s10950-008-9136-9

ORIGINAL ARTICLE

The reality of the scaling law of earthquake-source spectra?

Igor A. Beresnev

Received: 22 April 2008 / Accepted: 1 September 2008

Abstract Attempts to build a "constant-stress-drop" scaling of an earthquake-source spectrum have invariably met with difficulties. Physically, such a scaling would mean that the low-frequency content of the spectrum would control the high-frequency one, reducing the number of the parameters governing the time history of a shear dislocation to one. This is technically achieved through relationships of the corner frequency of the spectrum to the fault size, inevitably introduced in an arbitrary manner using a constant termed "stress drop". Throughout decades of observations, this quantity has never proved to be constant. This fact has fundamental physical reasons. The dislocation motion is controlled by two independent parameters: the final static offset and the speed at which it is reached. The former controls the low-frequency asymptote of the spectrum while the latter its high-frequency content. There is no physical reason to believe that the static displacement should predetermine the slip rate, which would be implied if the "stress drop" were constant. Reducing the two parameters to just one (the seismic moment or magnitude) in a "scaling law" has no strict justification; this would necessarily involve arbitrary assumptions about the relationship of one parameter to the other. This explains why the "constant-stress-drop" scaling in seismology has been believed in but never reconciled with the data.

Keywords Earthquake source · Scaling law · Seismic spectrum · Constant stress drop

I. A. Beresnev (✉)
Department of Geological & Atmospheric Sciences, Iowa State University, 253 Science I, Ames, IA 50011-3212, USA
e-mail: beresnev@iastate.edu

1 Introduction

The notion of a "scaling law" is widely used in engineering seismology, not lastly because of its practical expediency. Indeed, the Fourier spectrum of an earthquake seismogram is typically represented as a product of the source, path, and site terms (e.g., Boore 1983, Eq. 1). It is, therefore, extremely helpful if one could specify the entire source term by introducing one parameter, usually the seismic moment (or magnitude). This is the definition of scaling (Aki 1967, p. 1222).

It is clear even intuitively, though, that the low-frequency measure, such as the moment (or magnitude), can only provide limited information about the source process. The moment, by its definition, is determined by the final slip on the fault. The way the slip reaches its final state cannot be recovered from the value of the moment, and the more detailed slip history must be "recorded" in the higher-frequency content of the spectrum. Even this simple observation speaks against the possibility of a single low-frequency measure defining the entire shape of the spectrum. There have nevertheless been well-argued justifications of the spectral-scaling law. The question then is, "Are they really compatible with the observations and the physical reality?" Ultimately, "Is there a physical basis for the assumption that the entire source spectrum be a function of its low-frequency asymptote?" In addressing these questions, I first revisit the common assumptions behind the validity of the scaling law and then argue that the concept of scaling simply neglects the existence, even in the simplest case, of at least two independent parameters controlling the shape of the spectrum. It is, thus, not a viable model.

2 The "constant-stress-drop" scaling

For simplicity, I will assume a simple shear-dislocation model of an earthquake (Aki and Richards 1980, Fig. 4.4) and acknowledge the fact that an "ω^2-shaped" spectrum is typically observed, at least for small-to-moderate earthquakes. The displacement time history of a dislocation that radiates the "ω^2" spectrum is

$$u(t) = U[1 - (1 + t/\tau)\exp(-t/\tau)], \tag{1}$$

where U is the final displacement and τ is the parameter governing the speed at which the displacement reaches its final value (Beresnev and Atkinson 1997, Eq. 6). The far-field Fourier amplitude spectrum of particle displacement is then proportional to the spectrum of the time derivative of Eq. 1,

$$u_{\mathrm{FF}}(\omega) = CM_0\left[1 + (\omega/\omega_{\mathrm{c}})^2\right]^{-1}, \tag{2}$$

where C is a constant and M_0 is the seismic moment (Beresnev and Atkinson 1997, Eq. 11). The quantity

$$\omega_{\mathrm{c}} \equiv 1/\tau \tag{3}$$

is the "corner frequency" of the spectrum. It is clear from Eq. 2 that, in the low-frequency range ($\omega << \omega_{\mathrm{c}}$) the spectrum is controlled by M_0, whereas at high frequencies ($\omega >> \omega_{\mathrm{c}}$), it is controlled by the product $M_0\omega_c^2$. The high-frequency behavior cannot therefore be represented by the low-frequency measure M_0 alone.

A common assumption is then made that ω_c could be related to the moment, and the entire spectrum (2) then rendered dependent on M_0 only. The quantity ω_c, through Eq. 3, is the inverse of the characteristic time scale τ of the slip process. The slip duration T (rise time) at a given point on the fault can be hypothesized to be related to the total time of the rupture propagation across the fault, $T = CL/\beta$, where β is the shear-wave propagation speed, L is the characteristic dimension of the fault, and C is an unknown coefficient of proportionality (Kanamori and Anderson 1975, Eq. 10). We have assumed that the rupture propagates at a fraction of the shear-wave speed. One could then use an approximate (but by no means exact) equivalence of T and τ; equating them leads to a highly uncertain (even speculative) relation of the type

$$\omega_{\mathrm{c}} = K(\beta/L), \tag{4}$$

where K is another undetermined coefficient. Assigning any particular value to K would be meaningless, since there are three uncertain relationships built into it (Beresnev 2001, p. 398). Specifically, they are: (1) the slip duration at a point is proportional to the rupture-propagation time, (2) the rupture-propagation velocity is a percentage of the shear-wave velocity, and (3) the mathematical time scale τ characterizes the rise time. Although all of these relations are reasonable, neither of them has an exact, once-for-all prescribed form, begging caution in applying the resulting Eq. 4. Even putting an uncertainty level on such a relation does not seem to be possible, without the former being speculative. The hypotheses (1) and (2) have no reliable empirical constraints, and (3) is dealt with by a mere convention (Beresnev and Atkinson 1997). Such an uncertainty may be indirectly built into the observed scatter of the scaling constant "stress drop" discussed below, reaching three orders of magnitude, which is invariably found if experimental data are fit with a scaling model.

The equations of type (4) are known as Brune's relations (Brune 1970 Eq. 36; 1971).

The final step involves using the definition of the moment, $M_0 = \mu U A$, where μ is the shear modulus and A the fault area, and the quantity usually called the "stress drop",

$$\Delta\sigma = \mu U / L, \tag{5}$$

to transform Eq. 4 into the form

$$\omega_c = K\beta \left(\Delta\sigma / M_0\right)^{1/3}, \tag{6}$$

with an arbitrary coefficient K and the same uncertainty as in the original relation (4) (Beresnev 2001, Eq. 14). The equation of this type is referenced, for example, by Boore (1983, Eq. 5). Equation 6 presumably achieves the goal of transforming the spectrum (2) into the form depending only on the seismic moment, provided the quantity $\Delta\sigma$ and the coefficient K, which implicitly includes the three uncertain relationships, can all be considered constant. This is the essence of the "constant-stress-drop" scaling.

One can see, however, that building the scaling law in such a way is founded on shaky ground. First, we need to re-emphasize the speculative character of Eq. 6 based on that of Eq. 4. Second, the quantity $\Delta\sigma$ defined by either Eqs. 5 or 6, termed the "stress drop" but argued to bear little relevance to any real physical quantity (Boore 1983, p. 1868; Atkinson and Beresnev 1997; Beresnev 2001), typically exhibits enormous scatter, varying by at least three orders of magnitude (1–1,000 bars). Also, the measurements based on these two equations are not equivalent. If one uses definition (5) to determine the stress drop, the result cannot be applied to describe real source spectra, because the relationship (6) is uncertain. On the other hand, if one uses relationship (6) to calculate the stress drop from a real spectrum, the result will be highly variable (never constant), not only because of the uncertainty in Eq. 6 but also because this result will implicitly include the variability in the slip velocity (the parameter τ excluded from Eq. 6) on real faults. The results obtained in these two manners are typically used interchangeably, although they report different quantities. A great variability in the "observed" values should, thus, be expected.

Not surprisingly, the last three decades of observational seismology have proved the lack of any "constant-stress-drop" reality, if one is willing to allow an alternative interpretation of the facts. The results of comprehensive studies, which report stress-drop estimation using variations of Eq. 6 for dozens of earthquakes in a wide magnitude range, such as those of Abercrombie (1995), Hough and Dreger (1995), or Humphrey and Anderson (1994), emphasize this point. The scatter of the obtained values exceeds three orders of magnitude (Humphrey and Anderson 1994, their Fig. 7; Abercrombie 1995, her Fig. 11; Hough and Dreger 1995, their Figs. 6–7), even for a given moment. The error in the calculation of $\Delta\sigma$ from Eq. 6, stemming just from the uncertainty in the measurement of the corner frequency and inaccurate knowledge of the shear-wave velocity, is estimated as one order of magnitude by Hough and Dreger (1995, p. 1588) and cannot explain the cloud. As a result, the latter authors argue for a "real variability in stress drop" (Ibid.). We add that the arbitrary character of the relationship (6) is another (perhaps, the most significant) factor causing the scatter.

This simply means that the scaling relation resulting from Eq. 2 if the corner frequency in it is replaced by its proxy Eq. 6 is not a viable source model.

3 The physical reason for the lack of "scaling"

As we have seen, the attempts to build a "constant-stress-drop" scaling model of an earthquake-source spectrum have invariably met with difficulties. In our view, this is fundamentally related to the futility of the attempts to reduce both parameters that govern the shape of the spectrum (2), M_0 and ω_c, to one low-frequency measure. Physically, this is well understood. The shear dislocation is characterized by two well defined physical parameters: the final static offset and the speed at which it is approached. It would be difficult to justify why the final slip should necessarily predetermine the slip rate. Mathematically, this is reflected in the impossibility of building a rigorous relation of the type (6).

4 Conclusions

The same work that originally introduced the scaling law ended with its virtual denial. We read, “We shall probably have to assign different scaling laws to different environments. This implies that a single parameter, such as magnitude, cannot describe an earthquake even as a rough measure” (Aki 1967, p. 1230). I have aimed to prove that this conclusion has fundamental physical reasons. Even in the case of a point shear dislocation, the rupture-propagation effects set aside, the shape of the displacement spectrum of the far-field radiation is governed by two independent parameters. The low-frequency asymptote is controlled by the static dislocation through the moment M_0, and the high-frequency one is controlled by the parameter τ that is the characteristic time of the dislocation rise to its final value. The latter characterizes the speed at which the dislocation grows. There cannot be a physical justification to building a spectral-scaling law based on just one parameter, without forcing one to arbitrarily depend on the other. Two parameters, instead of one, should necessarily be invoked to characterize the observed spectra in order to remove the paradox of the “constant stress drop” never being constant. I have, thus, offered an alternative view at the facts which, instead of forcing them into the framework of scaling, explains the universally observed variability in the stress drops by admitting there is no constancy at all.

Acknowledgements This study was partially supported by Iowa State University. I am indebted to N. Deichmann and an anonymous reviewer for a critical insight into the paper.

References

Abercrombie RE (1995) Earthquake source scaling relationships from −1 to 5 M_L using seismograms recorded at 2.5-km depth. J Geophys Res 100:24015–24036

Aki K (1967) Scaling law of seismic spectrum. J Geophys Res 72:1217–1231

Aki K, Richards P (1980) Quantitative seismology. Theory and Methods. Freeman, San Francisco

Atkinson GM, Beresnev I (1997) Don’t call it stress drop. Seismol Res Lett 68:3–4

Beresnev IA (2001) What we can and cannot learn about earthquake sources from the spectra of seismic waves. Bull Seismol Soc Am 91:397–400

Beresnev IA, Atkinson GM (1997) Modeling finite-fault radiation from the ω^n spectrum. Bull Seismol Soc Am 87:67–84

Boore DM (1983) Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra. Bull Seismol Soc Am 73:1865–1894

Brune JN (1970) Tectonic stress and the spectra of seismic shear waves from earthquakes. J Geophys Res 75:4997–5009

Brune JN (1971) Correction. J Geophys Res 76:5002

Hough SE, Dreger DS (1995) Source parameters of the 23 April 1992 M 6.1 Joshua Tree, California, earthquake and its aftershocks: empirical Green’s function analysis of GEOS and TERRAscope data. Bull Seismol Soc Am 85:1576–1590

Humphrey JR Jr, Anderson JG (1994) Seismic source parameters from the Guerrero subduction zone. Bull Seismol Soc Am 84:1754–1769

Kanamori H, Anderson DL (1975) Theoretical basis of some empirical relations in seismology. Bull Seismol Soc Am 65:1073–1095